OBLIQUE SCATTERING AND COUPLING TO A SLIT COAXIAL CABLE: TM CASE

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Abstract—An exact series solution for the oblique scattering and coupling problems by an infinite, concentrically loaded slit cylinder with thickness is formulated for TM plane wave using mode matching technique. The scattered and penetrated fields are represented in terms of an infinite series of radial modes. By applying the appropriate boundary conditions, the coefficients of scattered and penetrated fields are obtained. Numerical results for the internal field are given, and these are in good agreement with some of the available published data.

1. INTRODUCTION

Electromagnetic penetration through and scattering from apertures are two basic problems in electromagnetic compatibility (EMC). When an electromagnetic interference (EMI) signal is incident upon the aperture, the estimation of the penetrated field into the aperture and the scattered field outside the aperture is important to evaluate the performance of various electrical systems and electromagnetic environments. A slotted circular cylinder is one of the most frequently investigated geometries in the area of scattering and radiation. Recently, the circular slotted cylinder has become the subject of extensive study due to its engineering applications in devices such as aperture and leaky wave antennas, microstrip transmission lines, microstrip antennas, composite missiles, and engine tubes of jet aircraft. The problem of scattering by an infinitely long slit cylinder has been treated in [1–13] using several techniques such as dual-series equations, singular integral equations, method of moment and characteristic currents theory, etc.
Most of the previous works deal with scattering and coupling problems of the infinitely thin slot when the plane of incidence is perpendicular to the slot axis. When the plane of incidence is at an arbitrary angle with respect to the slit axis, the problem becomes three-dimensional oblique incidence case and the scattering behavior is not well understood. In this paper, an exact series solution for the oblique scattering and coupling properties of an infinite, concentrically loaded slot cylinder with thickness shown in Fig. 1 is developed by using radial mode matching technique.

![Figure 1. Geometry of the problem.](image)

2. THEORETICAL FORMULATION

2.1 Field Representations

Consider a $TM_z$ plane wave at $\phi = \phi_i$ and $\theta = \theta_i$ illuminating a concentrical circular cylinder with a longitudinal slot as shown in Fig. 1. Throughout the paper, $e^{j\omega t}$ time harmonic factor is assumed and suppressed. In Region I ($\rho > b$), the total field is composed of
Oblique scattering and coupling

incident, reflected and scattered fields, and it may be represented as,

\[ E_z^I(\rho, \phi) = F(\theta_i) \sum_{n=-\infty}^{\infty} \left\{ j^n J_n(\kappa \rho) e^{-jn\phi_i} + A_n H^{(2)}_n(\kappa \rho) \right\} e^{jn\phi} \]

where

\[ F(\theta_i) = \sin \theta_i e^{jk_0 z \cos \theta_i} \]
\[ \kappa = k_0 \sin \theta_i \]

and \( J_n(\cdots) \) and \( H^{(2)}_n(\cdots) \) are Bessel function of the first kind and Hankel function of the second kind, respectively.

The transmitted fields in Region II \((a < \rho < b, 0 < \phi < \phi_0)\) and Region III \((\rho < a)\) may also be represented as,

\[ E_z^{II}(\rho, \phi) = F(\theta_i) \sum_{p=1}^{\infty} \left\{ B_p J_\mu(\kappa \rho) + C_p J'_\mu(\kappa \rho) \right\} \sin \mu \phi \]  \tag{2}
\[ E_z^{III}(\rho, \phi) = F(\theta_i) \sum_{n=-\infty}^{\infty} D_n G_n(\kappa \rho) e^{jn\phi}, \]  \tag{3}

respectively, where \( Y_\mu(\cdots) \) is Bessel function of the second kind of order \( \mu \), and \( \mu = p\pi/\phi_0, \ p = 1, 2, 3, \ldots \), and

\[ G_n(\kappa \rho) = \begin{cases} J_n(\kappa \rho) ; \text{without conducting core} \\ J_n(\kappa \rho) - \frac{J_n(\kappa c)}{Y_n(\kappa c)} Y_n(\kappa \rho) \end{cases} ; \text{with conducting core} \]  \tag{4}

where \( c \) is radius of the conducting core. Similarly, the corresponding \( \phi \) components of the H-fields are given by

\[ H_\phi^I(\rho, \phi) = \frac{F(\theta_i)}{j\omega \mu_0 \sin \theta_i} \sum_{n=-\infty}^{\infty} \left\{ j^n J'_n(\kappa \rho) e^{-jn\phi_i} + A_n H^{(2')}_n(\kappa \rho) \right\} e^{jn\phi} \]  \tag{5}
\[ H_\phi^{II}(\rho, \phi) = \frac{F(\theta_i)}{j\omega \mu_0 \sin \theta_i} \sum_{p=1}^{\infty} \left\{ B_p J'_\mu(\kappa \rho) + C_p Y'_\mu(\kappa \rho) \right\} \sin \mu \phi \]  \tag{6}
\[ H_\phi^{III}(\rho, \phi) = \frac{F(\theta_i)}{j\omega \mu_0 \sin \theta_i} \sum_{n=-\infty}^{\infty} D_n G'_n(\kappa \rho) e^{jn\phi} \]  \tag{7}

2.2 Matching Boundary Conditions

To determine the unknown coefficients \( A_n, B_p, C_p \) and \( D_n \) in Eqs. (1)–(7), it is necessary to match the boundary conditions of tangential E- and H-field continuities at \( \rho = a \) and \( \rho = b \).
First, the tangential E-field continuity at $\rho = a$ yields

$$
\sum_{n=-\infty}^{\infty} D_n G_n(\kappa \alpha)e^{in\phi} = \sum_{p=1}^{\infty} \left\{ B_p J_{\mu}(\kappa \alpha) + C_p Y_{\mu}(\kappa \alpha) \right\} \sin \mu \phi U
$$

where $U$ is 1 for $0 \leq \phi \leq \phi_0$ and zero elsewhere. Applying orthogonality condition of exponential function with respect to $\phi$ from 0 to $2\pi$ gives

$$
2\pi D_k G_k(\kappa \alpha) = \sum_{p=1}^{\infty} \left\{ B_p J_{\mu}(\kappa \alpha) + C_p Y_{\mu}(\kappa \alpha) \right\} \hat{f}_{k\mu}
$$

where

$$
\hat{f}_{k\mu} = \int_0^{\phi_0} e^{-jk\phi} \sin \mu \phi d\phi
$$

Second, the tangential H-field continuity at $\rho = a$ and $a < \phi < \phi_0$ also yields

$$
\sum_{n=-\infty}^{\infty} D_n G_n'(\kappa \alpha)e^{in\phi} = \sum_{p=1}^{\infty} \left\{ B_p J_{\mu}'(\kappa \alpha) + C_p Y_{\mu}'(\kappa \alpha) \right\} \sin \mu \phi
$$

Eq. (12) can be obtained by applying orthogonality condition of sine function with respect to $\phi$ from $0 \leq \phi \leq \phi_0$ in a similar manner.

$$
B_p J_{\mu}'(\kappa \alpha) + C_p Y_{\mu}'(\kappa \alpha) = \frac{2}{\phi_0} \sum_{n=-\infty}^{\infty} D_n G_n'(\kappa \alpha) f_{n\mu}
$$

where

$$
f_{n\mu} = \int_0^{\phi_0} e^{j\mu \phi} \sin \mu \phi d\phi
$$

Next, the tangential E-field continuity at $\rho = b$ yields

$$
\sum_{n=-\infty}^{\infty} \left\{ j^n J_n(\kappa b)e^{-jn\phi_1} + A_n H_n^{(2)}(\kappa b) \right\} e^{jn\phi} = \sum_{p=1}^{\infty} \left\{ B_p J_{\mu}(\kappa b) + C_p Y_{\mu}(\kappa b) \right\} \sin \mu \phi U
$$

(14)
Oblique scattering and coupling

Applying orthogonality condition of exponential function with respect to $\phi$ from $0$ to $2\pi$ gives

$$2\pi \left\{ \sum_{p=1}^{\infty} \left( B_p J_\mu(\kappa b) + C_p Y_\mu(\kappa b) \right) \right\} \hat{f}_{k\mu}$$

As a final step, the tangential H-field continuity at $\rho = b$ yields

$$\sum_{n=-\infty}^{\infty} \left\{ j^n J_n'(\kappa b) e^{-jn\phi_i} + A_n H_n^{(2)'(\kappa b)} \right\} e^{j\mu\phi}$$

$$\sum_{p=1}^{\infty} \left\{ B_p J_\mu'(\kappa b) + C_p Y_\mu'(\kappa b) \right\} \sin \mu\phi$$

Again, by applying orthogonality conditions of sine function with respect to $\phi$ from $0 \leq \phi \leq \phi_o$, one can obtain

$$B_p J_\mu'(\kappa b) + C_p Y_\mu'(\kappa b) = \frac{2}{\phi_o} \sum_{n=-\infty}^{\infty} \left\{ j^n J_n'(\kappa b) e^{-jn\phi_i} + A_n H_n^{(2)'(\kappa b)} \right\} f_{n\mu}$$

Solving for $B_p$ and $C_p$ from Eq. (12) and (17) gives,

$$B_p = \frac{2 Y_\mu'(\kappa b)}{\phi_o \Delta_\mu} \sum_{n=-\infty}^{\infty} D_n G_n'(\kappa a) f_{n\mu}$$

$$\quad - \frac{2 Y_\mu'(\kappa a)}{\phi_o \Delta_\mu} \sum_{n=-\infty}^{\infty} \left\{ j^n J_n'(\kappa b) e^{-jn\phi_i} + A_n H_n^{(2)'(\kappa b)} \right\} f_{n\mu}$$

$$C_p = -\frac{2 J_\mu'(\kappa b)}{\phi_o \Delta_\mu} \sum_{n=-\infty}^{\infty} D_n G_n'(\kappa a) f_{n\mu}$$

$$\quad + \frac{2 J_\mu'(\kappa a)}{\phi_o \Delta_\mu} \sum_{n=-\infty}^{\infty} \left\{ j^n J_n'(\kappa b) e^{-jn\phi_i} + A_n H_n^{(2)'(\kappa b)} \right\} f_{n\mu}$$

where

$$\Delta_\mu \equiv J_\mu'(\kappa a) Y_\mu'(\kappa b) - J_\mu'(\kappa b) Y_\mu'(\kappa a)$$

Substituting (18) and (19) into (9) and (15), the simultaneous equation for $A_n$ and $D_n$ can be obtained as following,

$$j^k J_k(\kappa b) e^{-jk\phi_i} + A_k H_k^{(2)}(\kappa b)$$
\[
\begin{align*}
D_k G_k(\kappa a) &= \sum_{n=-\infty}^{\infty} D_n G_n'(\kappa a) M_{nk} + \sum_{n=-\infty}^{\infty} \left\{ j^n J_n' (\kappa b) e^{-jn\phi_1} + A_n H_n^{(2)'}(\kappa b) \right\} Q_{nk} \\
&= \sum_{n=-\infty}^{\infty} D_n G_n'(\kappa a) I_{nk} + \sum_{n=-\infty}^{\infty} \left\{ j^n J_n' (\kappa b) e^{-jn\phi_1} + A_n H_n^{(2)'}(\kappa b) \right\} R_{nk}
\end{align*}
\]

where

\[
\begin{align*}
I_{nk} &= \frac{1}{\pi \phi_0} \sum_{p=1}^{\infty} \frac{J_\mu (\kappa a) Y_\mu'(\kappa b) - J'_\mu (\kappa b) Y_\mu (\kappa a)}{\Delta_\mu} f_{n\mu} \hat{f}_{k\mu} \\
R_{nk} &= -\frac{2}{\pi^2 \phi_0} \frac{1}{\kappa a} \sum_{p=1}^{\infty} \frac{f_{n\mu} \hat{f}_{k\mu}}{\Delta_\mu} \\
M_{nk} &= \frac{2}{\pi^2 \phi_0} \frac{1}{\kappa b} \sum_{p=1}^{\infty} \frac{f_{n\mu} \hat{f}_{k\mu}}{\Delta_\mu} \\
Q_{nk} &= \frac{1}{\pi \phi_0} \sum_{p=1}^{\infty} \frac{J_\mu (\kappa a) Y_\mu (\kappa b) - J'_\mu (\kappa b) Y_\mu'(\kappa a)}{\Delta_\mu} f_{n\mu} \hat{f}_{k\mu}
\end{align*}
\]

The above simultaneous equation may be rewritten as the following matrix form,

\[
\begin{pmatrix}
\Psi_1 \\
\Psi_2 \\
\Psi_3 \\
\Psi_4
\end{pmatrix}
\begin{pmatrix}
J_n' (\kappa b) e^{-jn\phi_1} + A_n H_n^{(2)'}(\kappa b)
\end{pmatrix}
= \begin{pmatrix}
\Gamma \\
0
\end{pmatrix}
\]

where \(A_n\) and \(D_n\) are column vectors and \(\Psi_1, \Psi_2, \Psi_3, \Psi_4,\) and \(\Gamma\) are matrices whose elements are

\[
\begin{align*}
\psi_{1,nk} &= Q_{nk} - \frac{H_n^{(2)}(\kappa b)}{H_n^{(2)'}(\kappa b)} \delta_{nk} \\
\psi_{2,nk} &= \frac{G_n' (\kappa a)}{G_n (\kappa a)} M_{nk} \\
\psi_{3,nk} &= -R_{nk} \\
\psi_{4,nk} &= \delta_{nk} - \frac{G_n' (\kappa a)}{G_n (\kappa a)} I_{nk} \\
\gamma_n &= -\frac{2j}{\pi \kappa b} \frac{j^n e^{-jn\phi_1}}{H_n^{(2)'}(\kappa b)}
\end{align*}
\]
Solving the above matrix for $A_n$ and $D_n$, one can obtain

$$A_n H_n^{(2)''}(\kappa b) = (\Psi_1 - \Psi_2 \Psi_4^{-1} \Psi_3)^{-1} \Gamma - j^n J_n'(\kappa b) e^{-jn\phi_i}$$

$$D_n G_n(\kappa a) = -\Psi_4^{-1} \Psi_3 (\Psi_1 - \Psi_2 \Psi_4^{-1} \Psi_3)^{-1} \Gamma$$

Once $A_n$ and $D_n$ are determined, it is possible to evaluate the coefficient $B_p$ and $C_p$ using (18) and (19).

### 2.3 Current on an Interior Wire

A basic quantity of great interest in electromagnetic pulse (EMP) studies and in aperture coupling applications is the total current induced on an interior wire. It is a measure how well a particular field that has penetrated into the interior of an object through an aperture has coupled to an interior load. For the present two-dimensional configuration, the total current per unit length along the wire is defined by the integral relation:

$$I_z = c \int_0^{2\pi} H_\phi^{III}(c, \phi) d\phi$$

Since Eq. (7) for $H_\phi$ is in the form of a Fourier expansion, this expression is reduced to one which depends only on the coefficient $D_o$ as following.

$$I_z = \frac{2\pi c F(\theta_i)}{j \omega \mu_0 \sin \theta_i} D_o G_\phi'(\kappa c)$$

### 3. NUMERICAL RESULTS

To check the accuracy of the results presented in this paper, the problem of electromagnetic penetration into a slotted circular coaxial cable with air dielectric is considered first for a plane wave incidence. The radii of the outer and inner conductor are 1.0 m and 0.1 m, respectively, and the slot thickness is $t = 1 \text{ cm}$. Fig. 2 shows the magnitude of the current induced on the inner conductor due to an incident electric field of 1 V/m for two different aperture sizes. The general pattern of our results is in excellent agreement with thin case [8, Fig. 3]. The result in Fig. 2 reflects the fact that the mean level of the induced current is larger for larger aperture as expected. Fig. 3 shows the magnitude of the current induced on the inner conductor of the coaxial
Figure 2. The magnitude of the total current induced on the inner conductor for $a = 1.0 \text{ m}$, $c = 0.1 \text{ m}$, $t = 1 \text{ cm}$ when a plane wave is incident on the aperture.

cable for three different thickness of shell when the radius of the inner conductor and aperture size are $c = 0.3 \text{ m}$ and $\phi_o = 30^\circ$, respectively. Comparing the results of Fig. 3 with those of Fig. 2 (for aperture size of $\phi_o = 30^\circ$) over the same frequency range, it is noted that the mean level of the current induced on the inner conductor increases with its radius. Also, an increasing the thickness of shell causes a decrease the current induced on the inner conductor and an increase the resonant peaks level of the current. Fig. 4 shows the magnitude of the current induced on the inner conductor of the coaxial cable for three different oblique incidences when the radius of the inner conductor is $c = 0.1 \text{ m}$ and the aperture size is $\phi_o = 30^\circ$. A decrease in $\theta_i$ causes an increase in resonance frequency. In Fig. 5, the effects of the aperture coupling due to different incident angles are shown by comparing contour plots of the electric field generated by an E-polarized plane wave with $\lambda = 1.138 \text{ m}$ incident upon a slit cylinder of $a = 1 \text{ m}$, $c = 0.1 \text{ m}$, $t = 1 \text{ cm}$ and $\phi_o = 30^\circ$. In Fig. 5(a), the field penetrates into the interior and excite the $TM_{21}$ mode for $\theta_i = 90^\circ$. In contrast, the field penetrates and excite the $TM_{11}$ mode for $\theta_i = 45^\circ$ in Fig. 5(b) and (c). For $\theta_i = 30^\circ$, the field does not penetrate deeply into the
Figure 3. The magnitude of the current induced on the inner conductor with $a = 1 \text{m}$, $c = 0.3 \text{m}$ and $\phi_0 = 30^\circ$ for three different slit thickness when a plane wave is incident on the aperture ($\phi_i = 15^\circ$, $\theta_i = 90^\circ$).

Figure 4. The magnitude of the current induced on the inner conductor with $a = 1 \text{m}$, $c = 0.1 \text{m}$, $t = 1 \text{cm}$ and $\phi_0 = 30^\circ$ for three different oblique incidences when a plane wave is incident on the aperture ($\phi_i = 15^\circ$).
Figure 5. Contour plots of the electric field generated by four different oblique incidences with $\lambda = 1.138 \text{ m}$, $c = 0.1 \text{ m}$, $t = 1 \text{ cm}$ and $\phi_o = 30^\circ$ (a) $\phi_i = 15^\circ$, $\theta_i = 90^\circ$ (b) $\phi_i = 15^\circ$, $\theta_i = 45^\circ$ (c) $\phi_i = -75^\circ$, $\theta_i = 45^\circ$ (d) $\phi_i = 15^\circ$, $\theta_i = 30^\circ$.

interior as shown in Fig. 5(d). This is because the incident field has a wavelength below cut-off for the wire/slit cylinder waveguide.

4. CONCLUSIONS
The new and exact formulation of TM wave oblique scattering and coupling problems of an infinite, concentrically loaded slot cylinder with thickness is investigated in this paper. The radial-mode matching technique is used to obtain the scattered and penetrated fields in a series form. The magnitude of the induced current on the inner conductor is increased as the size of the aperture and radius of the inner conductor becomes larger. It is shown that the thickness of the cylinder and incident angle also cause considerable changes in field penetrated into the
interior and mode excitation. Diffraction and coupling properties for a thick slot are also given for several cases. The accuracy of the present method is checked with the existing solution of an infinitely long and thin slot in the case of normal incidence ($\theta_i = 90^\circ$).

REFERENCES


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