TM Scattering by a Wedge with Concaved Edge

Jong-Won Yu and Noh-Hoon Myung

Abstract—TM scattering problem by a perfectly conducting wedge with concaved edge is formulated for a line source excitation using the mode-matching technique. The scattered and guided fields are represented in terms of an infinite series of radial waveguide modes with unknown coefficients. By applying the appropriate boundary conditions, the coefficients of scattered field are obtained. For small \( \kappa \alpha \), the diffraction coefficient of concaved edge is derived from the scattered field.

Index Terms—Electromagnetic scattering.

I. INTRODUCTION

The scattering by an E-polarized electromagnetic plane wave incident on a conducting wedge is well known and may be evaluated asymptotically as the sum of a geometrical optics term \( E_{x}^{\text{GO}} \) plus an edge-diffracted term \( E_{x}^{\text{diff}} \) as postulated by Keller [1]. Also, the effects of a physical edge (not perfectly sharp) have been extensively studied. Weiner and Borison [2] have divided an actual cone tip into ball-point tip, rounded tip, and concaved tip to calculate the radar cross section (RCS) of the cone edge. Similarly, we divide a physical wedge edge into cylinder-tip edge, rounded edge, and concaved edge. Scattering by a half plane with cylinder-tip edge has been investigated exactly the behavior of scattering by a wedge with concaved edge and with that of perfect edge.

II. THEORETICAL FORMULATION

A two-dimensional problem of a wedge of angle \( 2\pi - \phi_s \) with concaved edge is illustrated in Fig. 1. The radius of concaved edge is denoted by \( \alpha \). The time dependence \( e^{j\omega t} \) is assumed and omitted throughout. Consider a TM\(_1\) line source at \( (\rho_s, \phi_s) \) illuminating a wedge with concaved edge. The space outside the concaved edge is divided into two regions; region I lies outside the fictitious circle while region II is inside the circle, as designated in the figure. The total electric fields in region I \((\rho > \alpha, 0 < \phi < \phi_s)\) and II \((\rho < \alpha)\) may be represented as, respectively,

\[
E^{\text{i}}_{x}(\rho, \phi) = E^{\text{i}}_{x} \left\{ \sum_{\eta = 1}^{\infty} \left\{ s_{\eta} H_{\eta}^{(2)}(\rho \alpha) J_{\mu}(k \rho) + B_{\eta} H_{\eta}^{(2)}(k \rho) \right\} \sin \mu \phi U_{1} \right\},
\]

\[
E^{\text{II}}_{x}(\rho, \phi) = E^{\text{II}}_{x} \left\{ \sum_{\eta = 1}^{\infty} \left\{ s_{\eta} J_{\mu}(k \rho \alpha) H_{\eta}^{(2)}(k \rho) + B_{\eta} J_{\mu}(k \rho \alpha) \right\} \sin \mu \phi U_{1} \right\}.
\]

Fig. 1. Geometry of a wedge with concaved edge.

\[ E^{\text{II}}_{x}(\rho, \phi) = E^{\text{II}}_{x} \sum_{n = -\infty}^{\infty} A_{n} J_{n}(k \rho) e^{j n \phi} \]

where \( E^{\text{II}}_{x} = -\eta_{c} k I_{1}/4 \), \( \eta_{c} \) is intrinsic impedance, and \( I_{1} \) is the strength of the electric current filament and \( s_{\eta} = 4 \pi / \phi \alpha \sin \mu \phi \), \( \mu = p \pi / \phi \alpha \), \( p = 1, 2, 3 \cdots \), and \( U_{1} = 1 \) for \( 0 \leq \phi < \phi_{s} \) and zero for \( \phi_{s} \leq \phi < 2 \pi \) and \( H_{\eta}^{(2)} \) and \( J_{\eta}^{(2)} \) are Bessel function of \( \mu \)th order and the first kind and Hankel function of \( \mu \)th order and the second kind, respectively.

To determine unknown coefficients \( A_{n} \) and \( B_{\eta} \), it is necessary to match the boundary conditions of tangential E- and H-field continuities. From the tangential E- and H-field continuities at \( \rho = \alpha \), we obtain simultaneous equations for the scattered field. After solving simultaneous equations using the orthogonality condition and the Wronskian of Bessel function, we have that

\[
\sum_{\eta = -\infty}^{\infty} \left\{ s_{\eta} H_{\eta}^{(2)}(k \alpha) J_{\mu}(k \alpha) + B_{\eta} H_{\eta}^{(2)}(k \alpha) \right\} \delta_{\eta \phi} - \frac{2j}{\pi k \alpha} \sum_{\eta = -\infty}^{\infty} s_{\eta} H_{\eta}^{(2)}(k \alpha) I_{\eta \phi}
\]

where \( \delta_{\eta \phi} \) is the Kronecker delta

\[
I_{\eta \phi} = \frac{1}{\pi \delta_{\eta \phi}} \sum_{k = -\infty}^{\infty} \frac{J_{\mu}(k \alpha)}{J_{\mu}(k \alpha)} G_{k \nu} \hat{G}_{k \nu}, \quad \mu(\nu) = p(\eta) \pi / \phi \alpha
\]

\[
G_{k \nu} = \int_{0}^{\phi_{s}} e^{j k \phi} \sin \mu \phi d \phi, \quad \hat{G}_{k \nu} = \int_{0}^{\phi_{s}} e^{-j k \phi} \sin \mu \phi d \phi.
\]

Equation (3) can be solved numerically to obtain the constants \( B_{\eta} \).

The infinite series involved in the solution is convergent (which is illustrated in the results), therefore, it will be truncated after a certain number of terms which depend on the largest argument of the Bessel function (i.e., \( k \alpha \)). Once \( B_{\eta} \) is determined, it is possible to evaluate the coefficient \( A_{n} \).

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The authors are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Yisoung Gu, Taejon, 305 701 Korea.

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III. SCATTERED FIELD COMPUTATION

Plane wave excitation is obtained by letting the line source recede to infinity. To obtain the far-scattered field, the asymptotic expansion of the Hankel function for a large argument is employed together with the well-known approximation for the field diffracted by a sharp wedge backscattered field pattern. An increase in the pattern at $\pi/2 < \phi < \pi$ and a small variation in the pattern at $\phi < \pi/2$, $\phi > \pi$. Phase data presented in radians are continuous except for a step of $\pi$ radians at $\phi = \pi/2$ and $\pi$, which originates in the singular behavior of the asymptotic result for the sharp wedge.

This new diffraction coefficient allows extension of geometrical diffraction theory to targets exhibiting concaved edge as well as perfect edge.

IV. CONCLUSION

This new diffraction coefficient allows extension of geometrical diffraction theory to targets exhibiting concaved edge as well as perfect edge.

REFERENCES