주름진 원형 실린더 Cavity-Backed Aperture의 전자파 산란 해석

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Electromagnetic Scattering from Cylindrical Cavity-Backed Apertures with Inner or Outer Longitudinal Corrugations : The Case of E-Polarization

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Abstract— **The exact series solution for the scattering of electric line source from a cylindrical cavity-backed aperture with longitudinal corrugations is formulated by using mode matching technique. The scattered and penetrated fields are represented in terms of an infinite series of radial modes. By applying the appropriate boundary conditions, the coefficients of the scattered and penetrated fields are obtained.**

Index Terms—

I. Introduction

ELECTROMAGNETIC wave scattering from corrugated structures constitutes an interesting problem because of the many applications of these structures. Corrugated structures have been used for more than 20 years in radiating systems, in telecommunications, and in radioastronomy. For instance, they are employed to realize polarization-insensitive shields, reflector panels, stealth structures in RCS-related problem, and for horn antennas. Due to their practical importance, corrugated surfaces have been widely analyzed using analytical and numerical methods. Nevertheless, published works refer mostly to planar periodic structures [1],[2] or to cylindrical corrugated structures [3],[4]. The situation where the substructure of the corrugation is more complex has not been considered.

In this paper, the problem of scattering from a perfectly conducting cylindrical cavity-backed aperture with inner corrugations has been considered. The corrugations may be, partially or totally, loaded with dielectric materials. This modified canonical problem can be used, for instance, to control RCS of a cylindrical cavity-backed apertures(CBA), which is the most investigated geometry in the area of scattering [5]-[10], and to model corrugated reflector panels. The technique in this paper parallel those of [10], where the slit coaxial cable problem for an obliquely incidence plane-wave excitation is addressed.

II. Theoretical formulation

A. *Inner-corrugated Cavity-Backed Aperture*

The geometries analyzed in this paper are shown in Figs. 1 and 2. Consider a TM_z plane wave at $\phi = \phi_i$ illuminating

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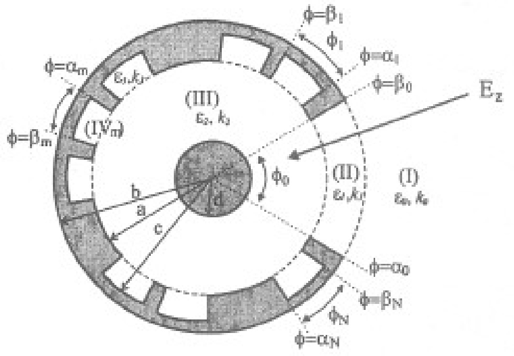

Fig. 1. E-wave scattering by an inner-corrugated cavity-backed aperture

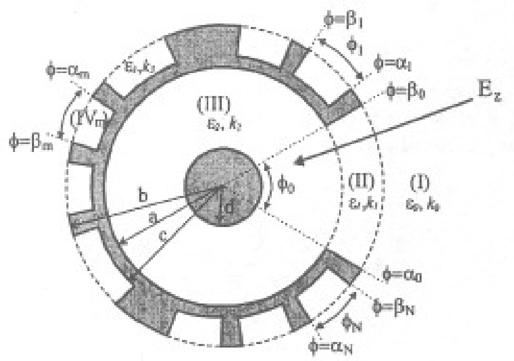

Fig. 2. E-wave scattering by an outer-corrugated cavity-backed aperture

a inner-corrugated cavity-backed aperture as shown in Fig. 1. Throughout the paper, the time dependence e^{jwt} is assumed and suppressed.

In region (I) ($\rho > b, 0 < \phi < 2\pi$), the total field is composed of incident, reflected and scattered fields, and it may be represented as,

$$E_z^{(I)}(\rho,\phi) = \sum_{n=-\infty}^{\infty} \left\{ j^n J_n(k_0\rho)e^{-jn\phi_i} + A_n H_n^{(2)}(k_0\rho) \right\} e^{jn\phi} \quad (1)$$

where k_0 is a wave number in free space and $J_n(\cdots)$ and

$H_n^{(2)}(\cdots)$ are Bessel function of the first kind and Hankel function the second kind, respectively.

The transmitted fields in region (II) ($a < \rho < b, \alpha_0 < \phi < \beta_0$) and region (III) ($\rho < a, 0 < \phi < 2\pi$) may also be represented as,

$$E_z^{(II)}(\rho,\phi) = \sum_{p=1}^{\infty} \left\{ B_p J_\mu(k_1\rho) + C_p Y_\mu(k_1\rho) \right\} \sin\mu(\phi - \alpha_0) \tag{2}$$

$$E_z^{(III)}(\rho,\phi) = \sum_{n=-\infty}^{\infty} D_n G_n(k_2\rho) e^{jn\phi} \tag{3}$$

respectively, where $Y_\mu(\cdots)$ is Bessel function of the second kind of order μ, and $\mu = p\pi/\phi_0, p = 1,2,3\cdots, \phi_0 = \beta_0 - \alpha_0$ and

$$G_n(k_2\rho) = \begin{cases} J_n(k_2\rho) & ;\text{ without core} \\ J_n(k_2\rho) - \frac{J_n(k_2 d)}{Y_n(k_2 d)} Y_n(k_2\rho) & ;\text{ with core} \end{cases} \tag{4}$$

where d is radius of the conducting core, and $k_n = k_0\sqrt{\mu_0\epsilon_n}$.

The total electric field of mth corrugation in region (IV) ($a < \rho < c, \alpha_m < \phi < \beta_m$) can be expressed as an infinite series of radial waveguide modes with unknown coefficients, i.e.

$$E_z^{(IV_m)}(\rho,\phi) = \sum_{q_m=1}^{\infty} E_{q_m} F_{\nu_m}(k_3\rho) \sin\nu_m(\phi - \alpha_m) \tag{5}$$

$$\begin{aligned} F_{\nu_m}(k_3\rho) &= J_{\nu_m}(k_3\rho) Y_{\nu_m}(k_3 c) - J_{\nu_m}(k_3 c) Y_{\nu_m}(k_3\rho) \\ \nu_m &= \frac{q_m \pi}{\phi_m}, \phi_m = (\beta_m - \alpha_m), q_m = 0,1,2,\cdots \end{aligned}$$

To determine unknown coefficients A_n, B_p , C_p, D_n and E_{q_m}, it is necessary to match the boundary conditions of tangential E- and H- field continuities at $\rho = a$ and $\rho = b$. Hence

$$\begin{aligned} B_p &= \frac{2}{\phi_0}\frac{k_2}{k_1}\frac{Y_\mu'(k_1 b)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) f_{n\mu}^0 \\ &- \frac{2}{\phi_0}\frac{k_0}{k_1}\frac{Y_\mu'(k_1 a)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} Z_n f_{n\mu}^0 \end{aligned} \tag{6}$$

$$\begin{aligned} C_p &= -\frac{2}{\phi_0}\frac{k_2}{k_1}\frac{J_\mu'(k_1 b)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) f_{n\mu}^0 \\ &+ \frac{2}{\phi_0}\frac{k_0}{k_1}\frac{J_\mu'(k_1 a)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} Z_n f_{n\mu}^0 \end{aligned} \tag{7}$$

$$E_{q_m} F_{\nu_m}'(k_3 a) = \frac{2}{\phi_m}\frac{k_2}{k_3} \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) f_{n\nu_m}^m \tag{8}$$

$$\begin{aligned} j^k J_k(k_0 b) e^{-jk\phi_i} + A_k H_k^{(2)}(k_0 b) = \\ \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) M_{kn} + \sum_{n=-\infty}^{\infty} Z_n Q_{kn} \end{aligned} \tag{9}$$

$$D_k G_k(k_2 a) = \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) I_{kn} + \sum_{n=-\infty}^{\infty} Z_n R_{kn} \tag{10}$$

where

$$\Delta_\mu = J_\mu'(k_1 a) Y_\mu'(k_1 b) - J_\mu'(k_1 b) Y_\mu'(k_1 a) \tag{11}$$

$$Z_n = j^n J_n'(k_0 b) e^{-jn\phi_i} + A_n H_n^{(2)'}(k_0 b) \tag{12}$$

and

$$\begin{aligned} I(k,n) &= \frac{1}{\pi\phi_0}\frac{k_2}{k_1}\sum_{p=1}^{\infty}\frac{JY_\mu^I}{\Delta_\mu} f_{n\mu}^0 f_{-k\mu}^0 \\ &+ \sum_{m=1}^{\infty}\sum_{q_m=1}^{\infty} \frac{1}{\pi\phi_m}\frac{k_2}{k_3}\frac{F_{\nu_m}(k_3 a)}{F_{\nu_m}'(k_3 a)} f_{n\nu_m}^m f_{-k\nu_m}^m \end{aligned} \tag{13}$$

$$R(k,n) = -\frac{2k_0}{\pi^2 k_1^2 a\phi_0}\sum_{p=1}^{\infty}\frac{f_{n\mu}^0 f_{-k\mu}^0}{\Delta_\mu} \tag{14}$$

$$M(k,n) = \frac{2k_2}{\pi^2 k_1^2 b\phi_0}\sum_{p=1}^{\infty}\frac{f_{n\mu}^0 f_{-k\mu}^0}{\Delta_\mu} \tag{15}$$

$$Q(k,n) = \frac{1}{\pi\phi_0}\frac{k_0}{k_1}\sum_{p=1}^{\infty}\frac{JY_\mu^Q}{\Delta_\mu} f_{n\mu}^0 f_{-k\mu}^0 \tag{16}$$

where

$$JY_\mu^I = J_\mu(k_1 a) Y_\mu'(k_1 b) - J_\mu'(k_1 b) Y_\mu(k_1 a) \tag{17}$$

$$JY_\mu^Q = J_\mu'(k_1 a) Y_\mu(k_1 b) - J_\mu(k_1 b) Y_\mu'(k_1 a) \tag{18}$$

$$f_{n\mu}^0 = \int_{\alpha_0}^{\beta_0} e^{jn\phi} \sin\mu(\phi - \alpha_0) d\phi \tag{19}$$

$$f_{n\nu_m}^m = \int_{\alpha_m}^{\beta_m} e^{jn\phi} \sin\nu_m(\phi - \alpha_m) d\phi \tag{20}$$

B. Outer-corrugated Cavity-Backed Aperture

We treat the second problem (for outer corrugation) in a similar way. The resultant infinite system of linear equations is formally the same as in the previous problem. It is rewritten for convenience as

$$\begin{aligned} B_p &= \frac{2}{\phi_0}\frac{k_2}{k_1}\frac{Y_\mu'(k_1 b)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) f_{n\mu}^0 \\ &- \frac{2}{\phi_0}\frac{k_0}{k_1}\frac{Y_\mu'(k_1 a)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} Z_n f_{n\mu}^0 \end{aligned} \tag{21}$$

$$\begin{aligned} C_p &= -\frac{2}{\phi_0}\frac{k_2}{k_1}\frac{J_\mu'(k_1 b)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} D_n G_n'(k_2 a) f_{n\mu}^0 \\ &+ \frac{2}{\phi_0}\frac{k_0}{k_1}\frac{J_\mu'(k_1 a)}{\Delta_\mu} \sum_{n=-\infty}^{\infty} Z_n f_{n\mu}^0 \end{aligned} \tag{22}$$

$$E_{q_m} F_{\nu_m}'(k_3 a) = \frac{2}{\phi_m}\frac{k_0}{k_3} \sum_{n=-\infty}^{\infty} Z_n f_{n\nu_m}^m \tag{23}$$

$$j^k J_k(k_0 b)e^{-jk\phi_i} + A_k H_k^{(2)}(k_0 b) = \sum_{n=-\infty}^{\infty} D_n G_n^{'}(k_2 a) M_{kn} + \sum_{n=-\infty}^{\infty} Z_n Q_{kn} \quad (24)$$

$$D_k G_k(k_2 a) = \sum_{n=-\infty}^{\infty} D_n G_n^{'}(k_2 a) I_{kn} + \sum_{n=-\infty}^{\infty} Z_n R_{kn} \quad (25)$$

and

$$I(k,n) = \frac{1}{\pi\phi_0}\frac{k_2}{k_1}\sum_{p=1}^{\infty}\frac{JY_\mu^I}{\Delta_\mu} f_{n\mu}^0(n,\mu) f_{-k\mu}^0 \quad (26)$$

$$R(k,n) = -\frac{2k_0}{\pi^2 k_1^2 a\phi_0}\sum_{p=1}^{\infty}\frac{f^0(n,\mu) f_{-k\mu}^0}{\Delta_\mu} \quad (27)$$

$$M(k,n) = \frac{2k_2}{\pi^2 k_1^2 b\phi_0}\sum_{p=1}^{\infty}\frac{f^0(n,\mu) f_{-k\mu}^0}{\Delta_\mu} \quad (28)$$

$$Q(k,n) = \frac{1}{\pi\phi_0}\frac{k_0}{k_1}\sum_{p=1}^{\infty}\frac{JY_\mu^Q}{\Delta_\mu} f_{n\mu}^0 f_{-k\mu}^0 + \sum_{m=1}^{\infty}\sum_{q_m=1}^{\infty}\frac{1}{\pi\phi_m}\frac{k_0}{k_3}\frac{F_{\nu_m}(k_3 b)}{F_{\nu_m}^{'}(k_3 b)} f_{n\nu_m}^m f_{-k\nu_m}^m \quad (29)$$

It is possible to solve the simultaneous equations (9),(10) or (24),(25) for unknown coefficients A_n and D_n. Once A_n and D_n are determined, it is possible to evaluate the coefficient B_p, C_p and E_{q_m} using (6), (7), (8) or (21),(22),(23) respectively.

III. Conclusions

The new and exact formulation of TM wave scattering and coupling problems of an infinite, concentrically loaded slot cylinder with corrugations is investigated in this paper. The radial mode matching technique is used to obtain the scattered and penetrated fields in a series form.

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